9. Mutual Induction

Mutual inductance occurs when the change in current in one inductor induces a voltage in another nearby inductor. It is the mechanism by which transformers work, but it can also cause unwanted coupling between conductors in a circuit. The mutual inductance, M, is the measure of the coupling between two inductors. In honor of Joseph Henry who developed the idea of inductance almost simultaneously with Faraday, the unit of inductance has been given the name henry (H): $1H = 1\frac{Wb}{A}$.

9.1. Theory

Current passing through a wire forming a loop produces a magnetic field. The magnitude of the magnetic field is directly proportional to the current. Doubling the number of loops doubles the magnitude of the magnetic field. Thus the magnitude of the magnetic field is directly proportional to the number of turns, N, in the current carrying coil of wire. The shape of the magnetic field line distribution will depend on the geometry of the loop or coil.

The magnetic flux, ϕ_m, through a loop of wire can be roughly understood as the number of magnetic field lines passing through the loop. Quantitatively, this flux is given by:

$$\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA, \tag{9.1}$$

and has units of **webers** ($1\,Wb = 1\,T\,m^2$). $\hat{n}$ is a unit vector perpendicular to dA. A weber is a line of induction in the mks system. B is the magnetic field in $\frac{Wb}{m^2}$.

If there are N loops in a tightly wound coil of wire the flux through the coil will be N times the flux through any single turn of the coil:

$$\phi_m = N\, \phi_{m,single}.$$

From now on we will always be talking about magnetic flux and will drop the subscript m.

If two coils of wire are in close proximity to one another the magnetic field produced by a current flowing in one coil (the **primary**) will produce a magnetic flux through the other (the **secondary**): ϕ_{21}. The subscript "21" denotes flux in the secondary caused by current in the primary. Since the magnetic field produced by the primary is directly proportional to I_1, the current in it, the total flux through the secondary will also be directly proportional to I_1:

$$\phi_{21} \propto I_1$$

We use M as the constant of proportionality to relate flux and current as shown in equation 9.2:

$$\phi_{21}[henrys] = M_{21} I_1 \tag{9.2}$$

where $M_{2,1}$ is the **mutual inductance** between the two coils. The mutual inductance will depend on the geometric characteristics of the two coils, their relative position and orientation.

A changing flux in a loop or coil induces an EMF $\mathcal{E}$ in the coil given by *Faraday's Law* in equation 9.3:

$$\mathcal{E} = -\frac{d\phi}{dt} \tag{9.3}$$

Therefore, a changing current in the primary will lead to an induced EMF in the secondary:

$$\mathcal{E}_2(t) = -M_{21}\frac{d}{dt}I_1(t)$$

If an alternating current with sinusoidal waveform:

$$I_1(t) = I_{1max}\cos\omega t$$

is passed through the primary coil, it will produce a sinusoidally varying flux $\phi_{21}(t) = M_{21}I_{1max}\cos wt$ in the secondary coil and a sinusoidally varying induced EMF (shown in Fig. 9.1):

$$\mathcal{E}_2(t) = -M_{21}\cdot I_{1max}\cdot\frac{d}{dt}\cos wt = \mathcal{E}_{2max}\sin\omega t$$

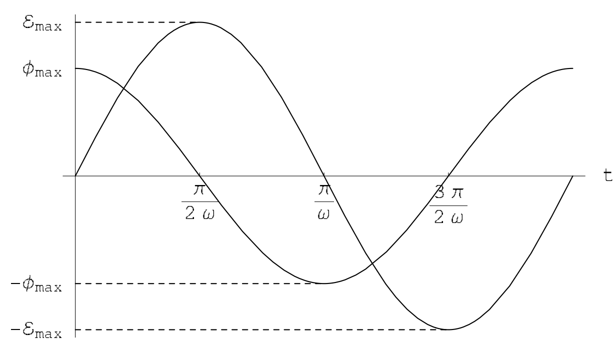

Figure 9.1.: Plot of magnetic flux and induced EMF

If we integrate the induced EMF over one half cycle we get:

$$\int_0^{\pi/\omega}\mathcal{E}\,dt = \int_0^{\pi/\omega}\mathcal{E}_{max}\sin\omega t\,dt = \frac{2\mathcal{E}_{max}}{\omega}.$$

Specifically in the case where the secondary is concerned,

$$\int_0^{\pi/\omega}\mathcal{E}_2(t)\,dt = \int_0^{\pi/\omega}\mathcal{E}_{2max}\sin\omega t\,dt = \frac{2\mathcal{E}_{2max}}{\omega}. \tag{9.4}$$

Alternatively, we could integrate the other side of Eq. (9.3) for the second coil:

$$\int_0^{\pi/\omega}\mathcal{E}_2(t)\,dt = -\int_0^{\pi/\omega}\frac{d\phi}{dt}\,dt = -\int_{\phi_{max}}^{-\phi_{max}}d\phi = 2\phi_{max} \tag{9.5}$$

where ϕ_{max} is the flux when the current is at its maximum value I_{max}.

Therefore the flux through the secondary coil at current I_{max} can be determined by measuring the angular frequency and the amplitude of the induced EMF waveform in the secondary coil:

$$\phi_{21} = \frac{\mathcal{E}_{2max}}{\omega}$$

The mutual inductance M_{21}between the coils can then be determined from a plot of ϕ as a function of I_{max}.

It should also be noted that current in the secondary coil would also lead to a magnetic flux through the primary coil:

$$\phi_{12} = M_{12}\,I_2.$$

Although it is not obvious, it can be shown that $M_{12} = M_{21}$. A rough value for the mutual inductance can actually determined from a single data point:

$$M_{12} = \frac{\phi_{12}}{I_2}.$$

9.2. Purpose

The objects of this experiment are to verify:

1. $\phi_{21} \propto I_1$
2. $\phi_{21} \propto N_1$
3. $\phi_{21} \propto N_2$
4. $\phi_{21} = \omega$

and to measure the mutual inductance of two coils. State all four of the relations above in words in your lab report.

9.3. Apparatus and Experiments

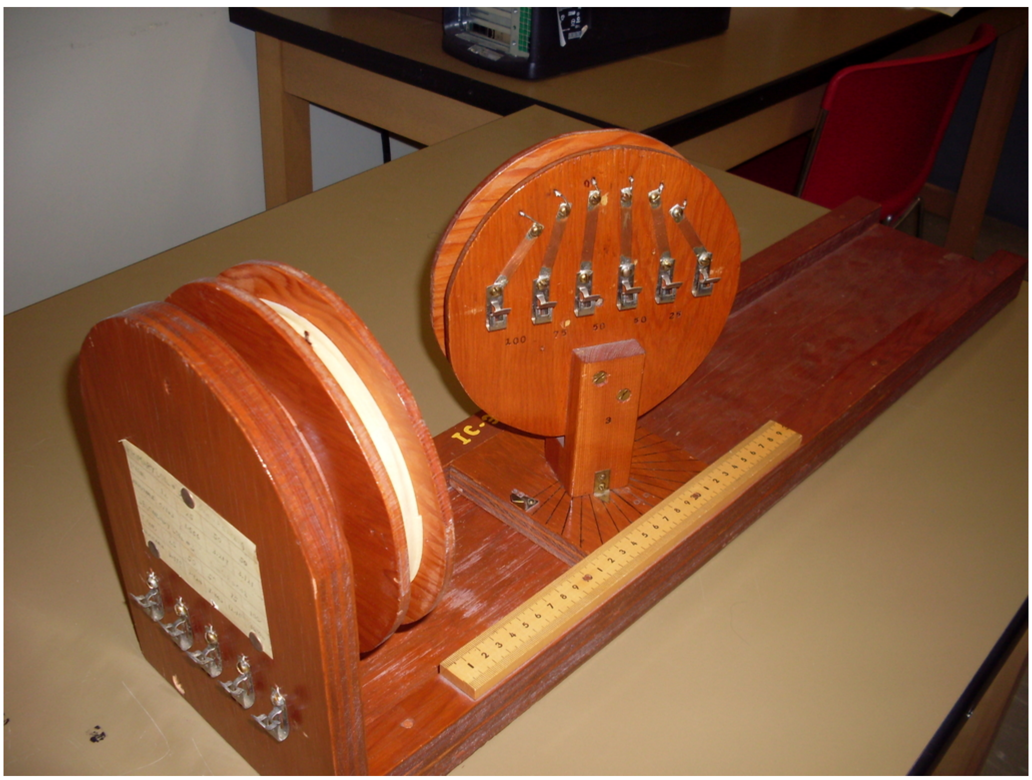

Figure 9.2.: Coil apparatus for mutual induction experiment.

The mutual inductance lab requires the use of two coils which are mounted on wooden forms. Figure 9.2 shows this apparatus. The construction of the coil apparatus allows for the rotation and translation of the secondary coil. The primary coil is fixed. Each coil has a number of taps which allow the experimenter to change N_1 and N_2. Additional equipment required consists on a Fluke DMM[1], and signal generator (aka function generator) and an oscilloscope with appropriate probes and connection wires.

[1] Other meters such as the Mastercraft meters do not read the current properly.

9.3.1. EXPERIMENT 1: $\phi_{21} \propto I_1$

Construct the circuit shown in figure 9.3. On the primary you must have the signal generator ground and the oscilloscope ground together. Select a sine wave output for the signal generator's output. Use a coil separation/orientation and turns value (both N_1 and N_2) which will maximize $\mathcal{E}_{2max}$. Monitor $\mathcal{E}_{1max}$ with channel 1 of the oscilloscope and I_1 with the DMM. Use a constant $\omega = 2\pi(200Hz)$ and confirm this is correctly set using the oscilloscope by measuring the period T. Change I_{1max} using the amplitude control on the function generator. Trigger the oscilloscope off channel 1.

Record all values set above in your report. The waveforms displayed by the scope may contain minor distortions from interference. The level of interference should not cause significant problems.

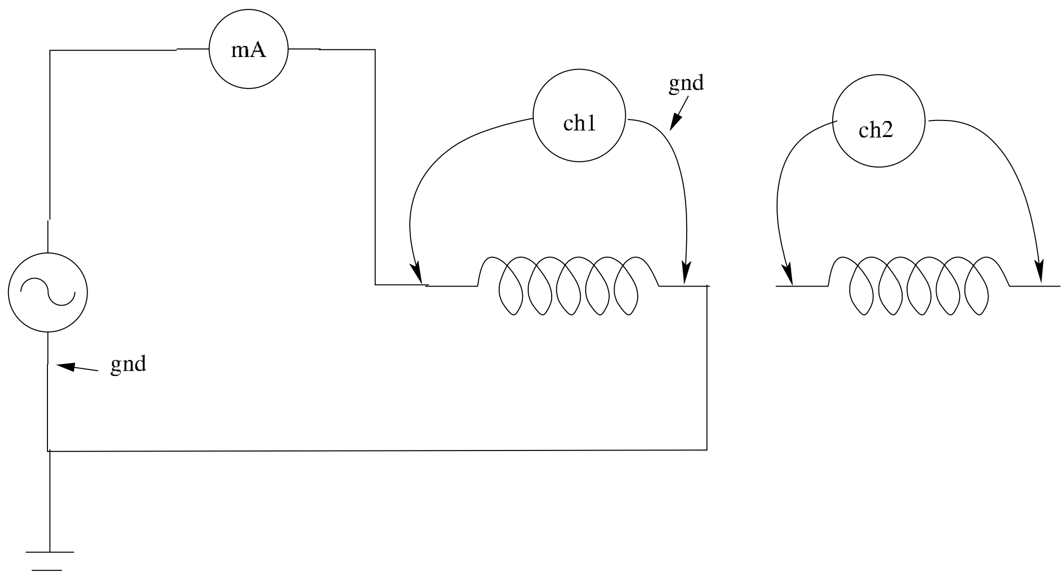

Figure 9.3.: Mutual Induction circuit.

1. Set $\mathcal{E}_{1p-p} = 1.6V_{p-p}$. Record the primary current I_1 value from the DMM and the induce voltage $\mathcal{E}_{2p-p}$.
2. Reduce $\mathcal{E}_{1p-p}$ to several values and record the primary current I_1 and the induced voltage $\mathcal{E}_{2p-p}$ each time.
3. Plot a graph of ϕ_{21} as a function of I_{1max} and determine the relation between them.

9.3.2. EXPERIMENT 2: $\phi_{21} \propto N_1$

A frequency of $f = 200Hz$ can be used for the rest of the experiment.

Using the same circuit from figure9.3, collect data for a graph to show that $\phi_{21} \propto N_1$. Again measure the voltage in the secondary $\mathcal{E}_{2max}$, since this is proportional to the flux ϕ_{21}. Vary N_1 through all possible values. ω, I_1 and N_2 must be kept constant[2]. Record all values used and measured. Plot a graph of ϕ_{21} as a function of N_1 and determine the relation between them.

Question:

How and why must I_1 be held constant for Experiment 9.3.2?

[2] Adjust your signal generator output if the current changes.

9.3.3. EXPERIMENT 3: $\phi_{21} \propto N_2$

This time ω, N_1 and I_1 must be held constant. ε_{2max} is measured again on channel 2. Use a current of about $70.0 mA_{rms}$ in the primary. Record all fixed values for your report. Change N_2 to several values for a plot of ϕ_{21} as a function of N_2.

9.3.4. EXPERIMENT 4: ω dependence

Since $\mathcal{E}_{2max} = \omega\phi_{21}$, changing the frequency of the driving voltage will affect the secondary peak voltage.

1. Maintain the current value at a constant value of about $90mA$. State your value in the report. Use the maximum N_1 and N_2 values as possible.
2. Change f to the following values and record the secondary voltage that results: 100, 200, 400, 800, 1200Hz.
3. Plot a graph to show that $\mathcal{E}_{2max} \propto \omega$.

Question

1. What would happen to electrical equipment if the hydro power supply suddenly doubled the frequency from *60Hz* to *120Hz*?

9.4. Further Experiments

The apparatus design allows the experimenter to change three more variables which affect $\frac{d\phi}{dt}$ in the secondary coil. These are d (the coil separation),θ (angle between the coils), and the material between the coils.[3].

Without taking data for graphs, use the apparatus for determine if the secondary voltage increases or decreases (or some other relation) with each variable. State you results in words in the lab report.

Also see Chapter 13.

[3] One can also test whether $M_{21} = M_{12}$